# HW 9 Help

**26.** **ORGANIZE AND PLAN** To calculate the mass of a certain volume if liquid, we need to know its density. The density of gasoline is given in Table 10.1.

*Known:* $V_l = 56$ L; $\rho_l = 680$ kg/m$^3$.

**SOLVE** The mass of gasoline is:

$$m = V_l \rho_l = (56\text{ L})(680\text{ kg/m}^3) = 38\text{ kg}$$

**REFLECT** Most liquids have densities similar to water, approximate 1 kg/L. A small difference in density is important, however, because it determines which liquid will float on top of the other if two liquids are placed in the same container (assuming the liquids don't mix).

**30.** **ORGANIZE AND PLAN** The stretch can be calculated from Equation 10.1 if we know the Young's modulus of copper. We can find the Young's modulus in Table 10.2.

*Known:* $L = 0.35$ m; $d = 7.0$ mm; $F = 1.2$ kN; $Y = 11 \times 10^{10}$ N/m$^2$.

**SOLVE** Equation 10.1 relates stress and strain:

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

Rewrite this equation to calculate the stretch:

$$\Delta L = \frac{F}{AY}L = \frac{F}{\frac{\pi}{4}d^2 Y}L = \frac{(1.2\text{ kN})}{\frac{\pi}{4}(7.0\text{ mm})^2(11\times 10^{10}\text{ N/m}^2)}(0.35\text{ m}) = 99\ \mu\text{m}$$

**REFLECT** The quantity $F/(AY)$ is dimensionless and equals the strain.

**33.** **ORGANIZE AND PLAN** The required force can be calculated from Equation 10.1 if we know the Young's modulus of steel. We can find the Young's modulus in Table 10.2. The mass to hang on the rod is the required force divided by *g*.

*Known:* $L = 1.5$ m; $d = 1.2$ mm; $\Delta L = 0.50$ mm; $Y = 20 \times 10^{10}$ N/m$^2$.

**SOLVE** Calculate the required force from Equation 10.1:

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$F = AY\frac{\Delta L}{L} = \tfrac{\pi}{4}d^2 Y\frac{\Delta L}{L} = \tfrac{\pi}{4}(1.2\text{ mm})^2(20\times 10^{10}\text{ N/m}^2)\frac{(0.50\text{ mm})}{(1.5\text{ m})} = 75\text{ N}$$

The mass to stretch the steel rod 0.50 mm is:

$$m = \frac{F}{g} = \frac{(75\text{ N})}{(9.80\text{ m/s}^2)} = 7.7\text{ kg}$$

**REFLECT** Equation 10.1 tells us that if we had placed the mass on top of the rod instead, we would have compressed the rod by the same amount, 0.50 mm. While this is mathematically correct, it would be difficult to do this in practice because the rod is very thin and would likely bend rather than compress.

**39. ORGANIZE AND PLAN** The pressure difference between sea level ( $P_0 = 1\text{ atm}$ ) and the ocean trench is given by Equation 10.4. The fractional volume change due to compression forces can be calculated from Equation 10.2, where the force per unit area is the pressure. We also need to know the bulk modulus of steel, and that is listed in Table 10.2.

*Known:* $h = 5.75\text{ km}$; $P_0 = 1\text{ atm}$; $\rho = 1000\text{ kg/m}^3$; $B = 16\times10^{10}\text{ N/m}^2$.

**SOLVE** (a) The pressure at a depth of 5.75 km is:

$$P = P_0 + \rho g h = (1\text{ atm}) + (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(5.75\text{ km}) = 56.5\text{ MPa}$$

(b) The fractional volume change of the steel spoon is given by Equation 10.2:

$$\frac{F}{A} = -B\frac{\Delta V}{V}$$

$$P = -B\frac{\Delta V}{V}$$

$$\frac{\Delta V}{V} = -\frac{P}{B} = -\frac{(5.65\times10^7\text{ N/m}^2)}{(16\times10^{10}\text{ N/m}^2)} = -3.53\times10^{-4}\text{ m}$$

**REFLECT** The spoon shrinks due to the compression forces.

**40. ORGANIZE AND PLAN** The pressure difference in a column of fluid with constant density equals the height of the column times the density times $g$.

*Known:* $h = 100\text{ m}$; $\rho = 1.28\text{ kg/m}^3$.

**SOLVE** For an altitude increase of 100 m, the air pressure decreases by:

$$\Delta P = \rho g h = (1.28\text{ kg/m}^3)(9.80\text{ m/s}^2)(100\text{ m}) = 1.25\text{ kPa}$$

**REFLECT** This is equivalent of 1.24% of an atmosphere. Is the assumption of constant air density a good one? For a small elevation change like 100 m, it is fairly good, but it does not hold for larger elevation changes. Passenger aircraft typically fly at altitudes of 10,000 m. If you assumed a constant air density, how much would the air pressure decrease for an altitude increase of 10,000 m?

**47. ORGANIZE AND PLAN** The pressure on each piston is the air pressure plus the applied force on that piston divided by the piston area. The pressures on the two pistons are equal when the system is in equilibrium. Because the pistons are at the same height, the air pressure is the same on both pistons.

*Known:* $A_1 = 0.50\text{ m}^2$; $A_2 = 5.60\text{ m}^2$; $F_1 = 2.0\text{ kN}$.

**SOLVE** The system is in equilibrium when:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

This means that the larger piston can support a force:

$$F_2 = \frac{A_2}{A_1}F_1 = \frac{(5.60\text{ m}^2)}{(0.50\text{ m}^2)}(2.0\text{ kN}) = 22\text{ kN}$$

i.e., it can support a mass:

$$m_2 = \frac{F_2}{g} = \frac{(22\text{ kN})}{(9.80\text{ m/s}^2)} = 2.3\times10^3\text{ kg}$$

**REFLECT** The first equation in our solution can be rewritten:

$$\frac{F_1}{F_2} = \frac{A_1}{A_2}$$

**50. ORGANIZE AND PLAN** The buoyant force is given by Archimedes's principle, Equation 10.5. We can calculate the buoyant force since we know from Table 10.1 that the density of air is $1.28 \text{ kg/m}^3$.

*Known:* $d = 17.5 \text{ cm}$; $\rho_{\text{fluid}} = 1.28 \text{ kg/m}^3$.

**SOLVE** First we calculate the volume of the balloon:

$$V = \frac{4\pi r^3}{3} = \frac{\pi d^3}{6} = \frac{\pi (17.5 \text{ cm})^3}{6} = 2.81 \times 10^3 \text{ cm}^3 = 2.81 \text{ L}$$

Then we use Equation 10.5 to calculate the buoyant force:

$$F_B = \rho_{\text{fluid}} g V = (1.28 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(2.81 \text{ L}) = 0.0352 \text{ N}$$

**REFLECT** This is a small force. The balloon will not be able to lift very much.

**53. ORGANIZE AND PLAN** Since the iceberg is not accelerating, the net force acting on the iceberg must be zero. The net force is the sum of the gravitational force and the buoyant force. That means the buoyant force is equal in magnitude to the gravitational force but in the opposite direction. Once we know the buoyant force we can use Archimedes's principle to calculate the volume of water displaced by the iceberg. Dividing this volume with the total volume of the iceberg we get the fraction of the iceberg's volume that is below the water line.

*Known:* $m = 6500 \text{ kg}$; $\rho_{\text{ice}} = 931 \text{ kg/m}^3$; $\rho_{\text{fluid}} = 1030 \text{ kg/m}^3$.

**SOLVE** (a) The buoyant force is:

$$F_B = -F_g = -(-mg) = mg = (6500 \text{ kg})(9.80 \text{ m/s}^2) = 63.7 \text{ kN}$$

(b) The volume of displaced water is calculated from Equation 10.5:

$$F_B = \rho_{\text{fluid}} g V$$

$$V = \frac{F_B}{\rho_{\text{fluid}} g} = \frac{(63.7 \text{ kN})}{(1030 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 6.311 \text{ m}^3$$

(c) The fraction of the iceberg's volume that is below that water line equals the volume of displaced water divided by the total volume of the iceberg:

$$\frac{V}{\frac{m}{\rho_{\text{ice}}}} = \frac{\rho_{\text{ice}} V}{m} = \frac{(931 \text{ kg/m}^3)(6.311 \text{ m}^3)}{(6500 \text{ kg})} = 90.4\%$$

**REFLECT** The fraction equals $\rho_{\text{ice}} / \rho_{\text{fluid}}$.

**66. ORGANIZE AND PLAN** The volume flow rate is the flow speed times the cross-sectional area.

*Known:* $d = 4.00 \text{ cm}$; $Q = 1.20 \times 10^{-4} \text{ m}^3\text{/s}$.

**SOLVE** The flow speed is:

$$v = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4} d^2} = \frac{(1.20 \times 10^{-4} \text{ m}^3\text{/s})}{\frac{\pi}{4}(4.00 \text{ cm})^2} = 0.0955 \text{ m/s}$$

**REFLECT** You can check that the formula is correct by making sure the units come out to meters per second (or distance per time).

**76. ORGANIZE AND PLAN** The lift force from the wings must oppose the gravitational force on the aircraft. Since we know the area of the wings, we can divide the force by the area to get the required difference between the pressure under the wings and the pressure over the wings. From Bernoulli's equation we can then calculate the airflow speed over the wings. We will use subscript 1 for quantities under the wings, subscript 2 over the wings.

*Known:* $m = 230{,}000$ kg; $v_1 = 75$ m/s; $A = 427$ m$^2$; $\rho = 1.28$ kg/m$^3$.

**SOLVE** The required lift force from the wings is:

$$F = mg = (230{,}000 \text{ kg})(9.80 \text{ m/s}^2) = 2.25 \text{ MN}$$

Dividing by the area of the wings we get the required pressure difference:

$$P_1 - P_2 = \frac{F}{A} = \frac{(2.25 \text{ MN})}{(427 \text{ m}^2)} = 5.28 \text{ kPa}.$$

The elevation difference between either side of the wing is very small and can be neglected. In this case Bernoulli's equation is:

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

Rewrite this equation and solve for the airflow speed over the wings:

$$v_2 = \sqrt{\frac{2}{\rho}(P_1 - P_2) + v_1^2} = \sqrt{\frac{2}{(1.28 \text{ kg/m}^3)}(5.28 \text{ kPa}) + (75 \text{ m/s})^2} = 1.2\times 10^2 \text{ m/s}$$

**REFLECT** From the equations above you can see that the faster the aircraft moves (the larger $v_1$ is), the smaller the required difference between $v_2$ and $v_1$ becomes. This fact is reflected in the design of an aircraft when you look at the shape of a wing's cross-section. Slow aircraft tend to have "fat" wings where the upper side of the wing is curved and thus longer than the lower side of the wing. Fast aircraft tend to have wings where the upper and lower sides are almost equal in length. When a fast aircraft has to fly slowly, for example when taking off and landing, it often requires the use of devices such as "flaps" that will modify the wing shape to look more like a wing shape of a slow aircraft.